**MT-205 Probability and Statistics**

**Assignment No# 2 Solution**

**Total Marks: 100**

**Q#1** In the manufacturing of a certain scientific instrument great importance is attached to the life of a particular critical component. This component is obtained in bulk from two sources, A and B, and in the course of inspection, the lives of 1000 of the components from each source are determined. The following frequency tables are obtained: (10 Marks)

|  |  |  |  |
| --- | --- | --- | --- |
| Source A | | Source B | |
| Life (hours) | No. of components | Life(hours) | No .of components |
| 1000-1020 | 40 | 1030-1040 | 339 |
| 1020-1040 | 96 | 1050-1050 | 136 |
| 1040-1060 | 364 | 1050-1060 | 25 |
| 1060-1080 | 372 | 1060-1070 | 20 |
| 1080-1100 | 85 | 1070-1080 | 130 |
| 1100-1120 | 43 | 1080-1090 | 350 |

1. Find Median and two quartiles for each group.

2. Find mean and Standard deviation for each source and compare them.

3. Which source do you think providing better quality of components and why? Note: answer this part by considering results of mean and standard deviation.

4. Find Quartile Deviation in terms of absolute and relative measure of dispersion.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Source A | | | | | Source B | | | | |
| Life (hours) | Frequency | Cumm.freq | x | Fx | Life(hours) | Frequency | Cumm. Freq | x | fx |
| 1000-1020 | 40 | 40 | 1010 | 40,400 | 1030-1040 | 339 | 339 | 1035 | 350,865 |
| 1020-1040 | 96 | 136 | 1030 | 98,880 | 1040-1050 | 136 | 475 | 1045 | 142120 |
| 1040-1060 | 364 | 500 | 1050 | 382200 | 1050-1060 | 25 | 500 | 1055 | 26375 |
| 1060-1080 | 372 | 872 | 1070 | 398040 | 1060-1070 | 20 | 520 | 1065 | 21300 |
| 1080-1100 | 85 | 957 | 1090 | 92650 | 1070-1080 | 130 | 650 | 1075 | 139750 |
| 1100-1120 | 43 | 1000 | 1110 | 47730 | 1080-1090 | 350 | 1000 | 1085 | 379750 |
|  | Total=1000 |  |  | Total: 1,059,900 |  | Total:1000 |  |  | Total: 10601160 |

1. Source A:

n=1000, h=20, l=1040, c=136

Median=l + (h/f) (n/2-c) = 1040 + (20/364) (500-136)

Median=1060

Q1=l + (h/f) (n/4-c) = 1040 + (20/364) (250-136)

Q1=1046.2

Q3=l+(h/f) (3n/4-c) = 1060 + (20/364) (750-500)

Q3=1073.7

Source B:

n=1000, h=10, l=1060, c=500, f=20

Median=l + (h/f) (n/2-c) = 1060 + (10/20) (500-500)

Median=1060

Q1=l + (h/f) (n/4-c) = 1030 + (10/339) (250-0)

Q1=1037.3

Q3=l+(h/f) (3n/4-c) = 1080 + (10/350) (750-650)

Q3=1082.8

1. Source A:

Mean= ∑fx /∑f = 1,059,900/1000

Mean=1060

Standard Deviation= =

Standard Deviation = 21.071

Source B:

Mean= ∑fx/∑f = 1,060,160/1000

Mean=1060.16

Standard Deviation=

Standard Deviation = 33515.7

1. Source A:

Absolute Q.D = = (1082.8 – 1037.3)/2

Absolute Q.D = 22.75

Relative Q. D= = (1082.8 – 1037.3) / (1082.8 + 1037.3)

Relative Q. D= 0.021

**Q#2** The number of foreign workers’ certificates for the New England states and the northwestern states is shown. Find the mean, median, and mode for both areas and compare the results.

Enrollment in local colleges, 2005 (10 Marks)

|  |  |
| --- | --- |
| **New England States** | **Northwest States** |
| 6768  3196  1112  819  1019  1795 | 1870  622  620  23  172  112 |
| Total= 14709 | Total=3419 |

1. Mean:

New England States:

Mean= = (6768 + 3196 + 1112 + 819+ 1019 + 1795) / 6

Mean= 2451.5

North-West States:

Mean== (1870 + 622 + 620 + 23 + 172 + 112) / 6

Mean= 2451.5

1. Median:

New England States:

Median = (n/2 + (n/2+1)) / 2 = (1112 + 1795) / 2

Mean= 1453.5

North-West States:

Median = (n/2 + (n/2+1)) / 2= (620 + 112) / 2

Median= 396

1. Mode:

New England States:

North-West States:

No mode

No mode

**Q#3 (a)** A group of 20 people played a PC game. The table below shows the frequency distribution of their scores. Given that the mean score is 5, find the value of find x. (5 Marks)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Score | 1 | 2 | 3 | x |
| No. of people | 2 | 5 | 7 | 6 |

Mean=

5 = (1x2 +2x5 + 3x7 + 6xx) /20

5 x 20 = 33 + 6x

x =11.16

**(b)** Given that the upper quartile is 110 and the median is 61. If the distribution is symmetric, find the value of the lower quartile.

Q3=110 Q2=61 Q1=?

Q3 - Q1 = Q2 - Q1

110 - Q1 = 61 – 110 => 110 + 49 = Q1 => Q1=159

**Q#4 (a)**The average number of days’ construction workers miss per year is 11. The standard deviation is 2.3. The average number of days’ factory workers miss per year is 8 with a standard deviation of 1.8. Which class is more variable in terms of days missed? (5 + 5 = 10 Marks)

C.V= (S.D / Mean) x 100 = (2.3 / 11) x 100 => C.V = 20.90

C.V= (S.D / Mean) x 100 = (1.8 / 8) x 100 => C.V = 22.5

**(b)** The average score on an English final examination was 85, with a standard deviation of 5; the average score on a history final exam was 110, with a standard deviation of 8. Which class was more variable?

C.V= (S.D / Mean) x 100 = (5 / 85) x 100 => C.V = 5.88

C.V= (S.D / Mean) x 100 = (8 / 110) x 100 => C.V = 7.27

**Q#5** The airborne speeds in miles per hour of 21 planes are shown. Find the approximate values that correspond to the given percentiles (10 Marks)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Class | 366-386 | 387-407 | 408-428 | 429-449 | 450-470 | 471-491 | 492-512 | 513-533 |  |
| Frequency | 4 | 2 | 3 | 2 | 1 | 2 | 3 | 4 | Total=21 |
| Cumm. Freq | 4 | 6 | 9 | 11 | 12 | 14 | 17 | 21 |  |

1. 9th percentile:

n=21 9n/100 = 1.89 l=365.5 h=21 c=0 f=4

P9= (l + h/f (9n/100) – c) = 365.5 + 21/4 (1.89 - 0) => P9 = 375

1. 20th percentile:

n=21 20n/100 = 5th l=386.5 h=21 c=4 f=2

P20= (l + h/f (20n/100) – c) = 386.5 + 21/2 (5 - 4) => P20 = 389

1. 45th percentile:

n=21 45n/100 = 21 l=428.5 h=21 c=9 f=2

P45= (l + h/f (45n/100) – c) = 428.5 + 21/2 (10 - 9) => P45 = 433

1. 60th percentile:

n=21 60n/100 = 12.6 ≈13th l=470.5 h=21 c=12 f=2

P60= (l + h/f (60n/100) – c) = 470.5 + 21/2 (12.6 - 12) => P60 = 477

1. 75th percentile:

n=21 75n/100 = 15.75 ≈ 16th l=491.5 h=21 f=3 c=14

P75= (l + h/f (9n/100) – c) = 491.5 + 21/3 (15.75 - 14) => P75 = 503.5

**Q#6 (a)** In an exam in statistics the mean grade of a group of 150 students was 78 and standard deviation was 8, in algebra the mean grade of a group was 73 and standard deviation was 7.6. In which subject was their Absolute dispersion and Relative dispersion high? (5 + 5 = 10 Marks)

n=150 Absolute: 8 > 7.5 (Statistics > Algebra)

Relative (Stats): (S.D / Mean) x 100 = (8 / 78) x 100 => C.V = 10.25

Relative (Algebra): (S.D / Mean) x 100 = (7.6 / 73) x 100 => C.V = 10.41

**(b)** An instructor grades exams, 20%; term paper, 30%; final exam, 50%. A student had grades of 83, 72, and 90, respectively, for exams, term paper, and final exam. Find the student’s final average. Use the weighted mean.

Mean = ∑wx / ∑w = ((20 x 83) + (30 x 72) + (50 x 90)) / 100 => Mean=83.2

**Q#7** Calculate Mean deviation about median from the following grouped data (5 Marks)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Class** | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |  |
| **Frequency** | 15 | 25 | 20 | 12 | 8 | 5 | 3 | Total=88 |
| **Cumm. Freq** | 15 | 40 | 60 | 72 | 80 | 85 | 88 |  |
| **x** | 15 | 25 | 35 | 45 | 55 | 65 | 75 |  |
| **|x –M|= |x-32|** | 17 | 7 | 3 | 13 | 23 | 33 | 43 |  |
| **f . |x - M|** | 255 | 175 | 60 | 156 | 184 | 165 | 129 | Total=1124 |

M = 30 + 10/20 (44 - 40) = 32

M.D= (∑f.|x-M|)/n = 1124/88 => M.D = 12.77

**Q#8** Use Q#7 data to calculate Mean deviation about mean (5 Marks)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Class** | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |  |
| **Frequency** | 15 | 25 | 20 | 12 | 8 | 5 | 3 | Total=88 |
| **Cumm. Freq** | 15 | 40 | 60 | 72 | 80 | 85 | 88 |  |
| **x** | 15 | 25 | 35 | 45 | 55 | 65 | 75 |  |
| **f . x** | 225 | 625 | 700 | 540 | 440 | 325 | 225 | Total=3080 |

Mean = ∑fx/∑f = 3080/ 88 => Mean = 35

**Q#9**  Team batting averages for major league baseball in 2015 are represented below. Find the variance and standard deviation for each league. Compare the results. (10 Marks)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **NL** | | | | | **AL** | | | | |
| Class | Freq | x | fx | fx2 | Class | Freq | x | fx | fx2 |
| 0.242–0.246 | 3 | 0.244 | 0.732 | 0.179 | 0.244-0.249 | 3 | 0.2465 | 0.7395 | 0.1823 |
| 0.247–0.251 | 6 | 0.249 | 1.494 | 0.372 | 0.250–0.255 | 6 | 0.2525 | 1.515 | 0.3825 |
| 0.252–0.256 | 1 | 0.254 | 0.254 | 0.065 | 0.256–0.261 | 2 | 0.2585 | 0.517 | 0.1336 |
| 0.257–0.261 | 11 | 0.259 | 2.849 | 0.738 | 0.262–0.267 | 1 | 0.2645 | 0.2645 | 0.0700 |
| 0.262–0.266 | 11 | 0.264 | 2.904 | 0.767 | 0.268–0.273 | 3 | 0.2705 | 0.8155 | 0.2195 |
| 0.267–0.271 | 1 | 0.269 | 0.269 | 0.072 | 0.274–0.279 | 0 | 0.2765 | 0 | 0 |
|  | ∑f = 33 |  | ∑fx = 8.502 | ∑fx2 = 2.193 |  | ∑f = 15 |  | ∑fx = 3.8475 | ∑fx2 = 0.9879 |

**NL:**

1. Variance:

Variance = (∑fx2 / ∑f) – (∑fx / ∑f )2 = (2.193/33) – (8.502/33)2

Variance = 7.80 x 10-5

1. Standard Deviation:

Standard Deviation = =

Standard Deviation = 8.83 x 10-3

**AL:**

1. Variance:

Variance = (∑fx2 / ∑f) – (∑fx / ∑f )2 = (0.9879/15) – (3.8475/15)2

Variance = 6.775 x 10-5

1. Standard Deviation:

Standard Deviation = =

Standard Deviation = 8.23 x 10-3

**Q#10** The number of observers in the Frog watch USA program (a wildlife conservation program dedicated to helping conserve frogs and toads) for the top 10 states with the most observers is 484, 483, 422, 396, 378, 352, 338, 331, 318, and 302. The top 10 states with the most active watchers list these numbers of visits: 634, 464, 406, 267, 219, 194, 191, 150, 130, and 114. Find the mean, mode, median and midrange for the data. Compare the measures of central tendency for these two groups of data. (10 Marks)

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Observers** | 484 | 483 | 422 | 396 | 378 | 352 | 338 | 331 | 318 | 302 | Total=3804 |
| **Visits** | 634 | 464 | 406 | 267 | 219 | 194 | 150 | 130 | 114 | 191 | Total=2769 |

For Observers:

1. Mean:

Mean = ∑X/N = 3804/10 => Mean = 380.4

1. Median:

Median = (n/2 + (n/2+1)) / 2 = (332+378) /2 => Median = 365

1. Mode:

No mode

1. Midrange:

Midrange = (484 + 302) / 2 => Midrange = 393

For Visits:

1. Mean:

Mean = ∑X/N = 2769/10 => Mean = 276.9

1. Median:

Median = (n/2 + (n/2+1)) / 2 = (194+219) /2 => Median = 206.5

1. Mode:

No mode

1. Midrange:

Midrange = (114 + 634) / 2 => Midrange = 374

**Q#11** The normal daily high temperatures (in degrees Fahrenheit) in January for 10 selected cities are as follows. 50, 37, 29, 54, 30, 61, 47, 38, 34, 61 The normal monthly precipitation (in inches) for these same 10 cities is listed here. 4.8, 2.6, 1.5, 1.8, 1.8, 3.3, 5.1, 1.1, 1.8, 2.5**.** Find the variance, standard deviation, range and coefficient of variance. Which is more variable? (10 Marks)

For the Temperature:

|  |  |
| --- | --- |
| **X** | **X2** |
| 29 | 841 |
| 30 | 900 |
| 34 | 1156 |
| 37 | 1369 |
| 38 | 1444 |
| 47 | 2209 |
| 50 | 2500 |
| 54 | 2916 |
| 61 | 3721 |
| 61 | 3721 |
| Sum= 441 | Sum=20,777 |

1. Range = 61 – 29 = 32 ii) Variance:

Variance = (∑x2 / n) – (∑x / n )2

= (20,777/10) – (441/10)2 => Variance = 132.89

1. Standard Deviation:

Standard Deviation = =

Standard Deviation = 11.52

1. Coefficient of Variance:

CVar = (S.D / Mean) x 100 = (11.52 / 44.1) x 100 => CVar = 26.12%

For the Precipitation:

|  |  |
| --- | --- |
| **X** | **X2** |
| 4.8 | 23.04 |
| 2.6 | 6.76 |
| 1.5 | 2.25 |
| 1.8 | 3.24 |
| 1.8 | 3.24 |
| 3.3 | 10.89 |
| 5.1 | 26.01 |
| 1.1 | 1.21 |
| 1.8 | 3.24 |
| 2.5 | 6.25 |
| Sum= 26.3 | Sum=86.13 |

1. Range = 5.1 – 1.1 = 4.0 ii) Variance:

Variance = (∑x2 / n) – (∑x / n )2

= (86.13/10) – (26.3/10)2 => Variance = 1.6961

1. Standard Deviation:

Standard Deviation = =

Standard Deviation = 1.30

1. Coefficient of Variance:

CVar = (S.D / Mean) x 100 = (1.30 / 2.63) x 100 => CVar = 49.51%

Precipitation is more variable since CVar is higher.

**Q#12** The costs of three models of helicopters are shown here. Find the weighted mean of the costs of the models. (5 Marks)

|  |  |  |
| --- | --- | --- |
| Model | Number sold | Cost |
| Sun scraper | 9 | $427,000 |
| Sky coaster | 6 | $365,000 |
| High-flyer | 12 | $725,000 |
|  | Total = 27 |  |

Mean = ∑wx / ∑w = ((9 x 427,000) + (6 x 365,000) + (12 x 725,000)) / 27

Mean = 545,666.6